Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 5, 2004*
Revised: *March 10, 2004*
Accepted: *March 10, 2004*

Consistent deformations of $[p, p]$–type gauge field theories

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Abstract: Using BRST-cohomological techniques, we analyze the consistent deformations of theories describing free tensor gauge fields whose symmetries are represented by Young tableaux made of two columns of equal length p, $p > 1$. Under the assumptions of locality and Poincaré invariance, we find that there is no consistent deformation of these theories that non-trivially modifies the gauge algebra and/or the gauge transformations. Adding the requirement that the deformation contains no more than two derivatives, the only possible deformation is a cosmological-constant-like term.

Keywords: Gauge Symmetry, BRST Symmetry.

* "Aspirant du F.N.R.S., Belgium".

Contents

1. Introduction

Recently, there has been a surge of interest in the study of theories for gauge fields transforming in "exotic" irreducible representations of the general linear group $GL(D)$ [1]–[11]. Such gauge fields are said to have mixed symmetries since they are neither completely symmetric nor completely antisymmetric. They are believed to play an important role in the description of tensionless string theories, where the higher-spin excitation modes become all massless [13, 14] (note that, in the context of string field theories, the "exotic" higher spin fields naturally appear in *reducible* representations of $GL(D)$).

In this paper we investigate the possible consistent deformations of free theories whose gauge fields $C_{\mu_1\ldots\mu_p|\nu_1\ldots\nu_p}$ possess the Young symmetry type[1] $[p,p]$, with $p>1$ but otherwise

[1]An irreducible representation of the general linear group $GL(D)$ is denoted by $[c_1, c_2, \ldots, c_L]$, where c_i indicates the number of boxes in the ith column of the Young tableau characterizing the corresponding irreducible representation. A p-form gauge field is then denoted by $[p]$ whereas a rank-s symmetric tensor is denoted by $[\underbrace{1,1,\ldots,1}_{\text{s times}}]$.

arbitrary:

$$C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p} \sim \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \vdots & \vdots \\ \hline p & p \\ \hline \end{array} \,.$$

The case $p = 1$ corresponds to linearized gravity and is treated in ref. [15]. We determine all the consistent, local interactions that these free theories possibly admit. By "consistent" we mean that the deformation is smooth in a formal deformation parameter g and reduces to the original theory in the free limit where $g = 0$.

Our approach is based on the BRST-cohomology-antifield formalism [16], in which consistent deformations of the action appear as deformations of the solutions of the master equation that preserve the master equation. The first-order consistent deformations define cohomological classes of the BRST differential at ghost number zero. We will not devote much time to the explanation of this technique; for more details and references, see e.g. [15] where the same BRST-antifield tools are used in the context of linearized Einstein theory.

Our result is that the free theory for the exotic field $C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p}$ admits no consistent local deformation that is compatible with Poincaré invariance and that non-trivially modifies the gauge algebra and/or the gauge transformations. This result holds without any assumption on the number of derivatives in the expressions for the gauge transformations and the interaction terms. Moreover, it is not required that the deformed gauge symmetries should close off-shell. If we demand that the deformations of the free lagrangian should not contain more than two derivatives of the fields (i.e., the allowed interaction terms under consideration schematically read "$C \cdots C$", "$C \cdots C \cdot \partial C$", "$C \cdots C \cdot \partial^2 C$", or "$C \cdots C \cdot \partial C \cdot \partial C$"), then the only possible deformation is a cosmological-constant-like term $a_0 = \Lambda C^{[p]}$, where $C^{[p]}$ denotes the p–th trace of the field $C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p}$.

2. The free theory

We work in arbitrary spacetime dimension D, only taken to be strictly greater than $2p+1$, so that the theory describes local degrees of freedom.

The free lagrangian for the gauge field $C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p}$ reads

$$\mathcal{L}_0 = (-)^p \frac{(p+1)}{2} \, \delta^{[\rho_1 \ldots \rho_p \mu_1 \ldots \mu_{p+1}]}_{[\nu_1 \ldots \nu_p \sigma_1 \ldots \sigma_{p+1}]} \, T_{\sigma_1 \ldots \sigma_{p+1}|\rho_1 \ldots \rho_p} \, T_{\mu_1 \ldots \mu_{p+1}|\nu_1 \ldots \nu_p} \,, \tag{2.1}$$

where T is the tensor

$$T_{\mu_1 \ldots \mu_{p+1}|\nu_1 \ldots \nu_p} = \partial_{[\mu_1} C_{\mu_2 \ldots \mu_{p+1}]|\nu_1 \ldots \nu_p} \,, \tag{2.2}$$

the square brackets $[\ldots]$ denote antisymmetrization with global weight one and $\delta^{\mu_1 \ldots \mu_n}_{\nu_1 \ldots \nu_n} \equiv \delta^{\mu_1}_{\nu_1} \ldots \delta^{\mu_n}_{\nu_n}$.

The gauge transformations are

$$\delta_\alpha C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p} = \alpha_{\mu_1 \ldots \mu_p|[\nu_1 \ldots \nu_{p-1}, \nu_p]} + \alpha_{\nu_1 \ldots \nu_p|[\mu_1 \ldots \mu_{p-1}, \mu_p]} \,, \tag{2.3}$$

where $\alpha_{,\nu} \equiv \partial_\nu \alpha$ and $\alpha_{\mu_1 \ldots \mu_p|\nu_1 \cdots \nu_{p-1}}$ is an arbitrary $[p, p-1]$-tensor field, that thus possesses the following algebraic properties:

$$\begin{aligned} \alpha_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_{p-1}} &= \alpha_{[\mu_1 \ldots \mu_p]|\nu_1 \ldots \nu_{p-1}} = \alpha_{\mu_1 \ldots \mu_p|[\nu_1 \ldots \nu_{p-1}]}\,, \\ \alpha_{[\mu_1 \ldots \mu_p|\mu_{p+1}]\nu_2 \ldots \nu_{p-1}} &= 0\,. \end{aligned} \tag{2.4}$$

To obtain a gauge-invariant object, one must take two derivatives of the field. The tensor

$$K_{\mu_1 \ldots \mu_{p+1}|\nu_1 \ldots \nu_{p+1}} = \partial_{[\mu_1} C_{\mu_2 \ldots \mu_{p+1}]|[\nu_1 \ldots \nu_p, \nu_{p+1}]} \tag{2.5}$$

is easily verified to be gauge invariant; moreover its vanishing implies that $C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p}$ is pure-gauge [4]. The most general gauge invariant object depends on the field $C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p}$ and its derivatives only through the "curvature" $K_{\mu_1 \ldots \mu_{p+1}|\nu_1 \ldots \nu_{p+1}}$ and its derivatives.

The equations of motion read

$$0 = G^{\mu_1 \ldots \mu_p|}{}_{\nu_1 \ldots \nu_p} \equiv (-)^{p+1}(p+1)\, \delta^{[\rho_1 \ldots \rho_{p+1} \mu_1 \ldots \mu_p]}_{[\nu_1 \ldots \nu_p \sigma_1 \ldots \sigma_{p+1}]} K_{\sigma_1 \ldots \sigma_{p+1}|\rho_1 \ldots \rho_{p+1}}\,. \tag{2.6}$$

They are equivalent to

$$\eta^{\mu\nu} K_{\mu\mu_1 \ldots \mu_p|\nu\nu_1 \ldots \nu_p} = 0\,. \tag{2.7}$$

Because the action is gauge-invariant, the equations of motion satisfy the "Bianchi identities"

$$\partial_{\mu_1} G^{\mu_1 \ldots \mu_p|}{}_{\nu_1 \ldots \nu_p} \equiv 0\,. \tag{2.8}$$

An easy way to check these identities is to observe that one has

$$G^{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p} \equiv \partial_{\alpha\beta} M^{\alpha\mu_1 \ldots \mu_p|\beta\nu_1 \ldots \nu_q}\,, \tag{2.9}$$

where the $[p+1, p+1]$-type tensor $M^{\alpha\mu_1 \ldots \mu_p|\beta\nu_1 \ldots \nu_q}$ is given by

$$M^{\alpha\mu_1 \ldots \mu_p|\beta\nu_1 \ldots \nu_p} = (-)^{p+1}(p+1)\, \delta^{[\rho_1 \ldots \rho_p \alpha \mu_1 \ldots \mu_p]}_{[\nu_1 \ldots \nu_p \beta \sigma_1 \ldots \sigma_p]} C_{\sigma_1 \ldots \sigma_p|\rho_1 \ldots \rho_p}\,. \tag{2.10}$$

The gauge symmetries (2.3) are reducible. Indeed,

$$\delta_{\tilde{\alpha}^{(1)}} C_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_p} \equiv 0 \tag{2.11}$$

when

$$\tilde{\alpha}^{(1)}_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_{p-1}} = \tilde{\alpha}^{(2)}_{\mu_1 \ldots \mu_p|[\nu_1 \ldots \nu_{p-2}, \nu_{p-1}]} + \frac{p}{2}\, \tilde{\alpha}^{(2)}_{\nu_1 \ldots \nu_{p-1}[\mu_p|\mu_1 \ldots \mu_{p-2}, \mu_{p-1}]}\,, \tag{2.12}$$

where $\tilde{\alpha}^{(2)}_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_{p-2}}$ is an arbitrary field of Young symmetry type $[p, p-2]$. For $p > 2$, there is further local reducibility. There are altogether $p-1$ reducibilities, involving arbitrary $[p, p-i]$-type fields $\tilde{\alpha}^{(i)}_{\mu_1 \ldots \mu_p|\nu_1 \ldots \nu_{p-i}}$, $2 \leq i \leq p$.

3. Deforming the free theory

The problem of introducing (smooth) consistent interactions is that of simultaneously deforming the free lagrangian, the gauge transformations and the reducibility conditions:

$$\mathcal{L} = \mathcal{L}_0 + g\mathcal{L}_1 + g^2\mathcal{L}_2 + \cdots \,, \tag{3.1}$$

$$\delta_\alpha C = (2.3) + g\,\delta_\alpha^{(1)} C + g^2\,\delta_\alpha^{(2)} C + \cdots \,, \tag{3.2}$$

$$\delta_\alpha \tilde{\alpha}^{(i)} = \delta_\alpha^{(0)} \tilde{\alpha}^{(i)} + g\,\delta_\alpha^{(1)} \tilde{\alpha}^{(i)} + g^2\,\delta_\alpha^{(2)} \tilde{\alpha}^{(i)} + \cdots \,, \tag{3.3}$$

in such a way that the deformed lagrangian is invariant under the deformed gauge transformations, the latter obeying the new reducibility conditions.

We shall impose the further requirement that the first order vertex $\mathcal{L}_1$ be Poincaré-invariant and local. Under this sole condition (together with consistency), we show that the gauge algebra together with the gauge transformations cannot be deformed.

Adding the requirement that the deformed lagrangian should not contain more than two derivatives, we further show that, except for a cosmological-constant-like term, there is no deformation that would only modify the free lagrangian, leaving the gauge structure unchanged.

3.1 BRST differential and field content

As shown in [16], the first-order consistent local interactions correspond to elements of the cohomology $H^{D,0}(s|d)$ of the BRST differential s modulo the total derivative d, in maximum form-degree D and in ghost number 0. That is, one must compute the general solution of the cocycle condition

$$sa + db = 0\,, \tag{3.4}$$

where a is a D-form of ghost number zero and b a $(D-1)$-form of ghost number one, with the understanding that two solutions a and a' of (3.4) that differ by a trivial solution

$$a' = a + sm + dn \tag{3.5}$$

should be identified as they define the same interactions up to field redefinitions. The cocycles and coboundaries $a, b, m, n, \ldots$ are functions of the field variables (including ghosts and antifields) and their derivatives, up to some finite order in the derivatives. They are called "local functions". Given a non-trivial cocycle a of $H^{D,0}(s|d)$, the corresponding first-order interaction vertex $\mathcal{L}_1$ is obtained by setting the ghosts equal to zero.

Since the theory at hand is a free theory, the BRST differential takes the simple form

$$s = \delta + \gamma\,, \tag{3.6}$$

where δ is the Koszul-Tate differential and γ is the exterior derivative along the gauge orbits. A grading is associated to each of these differentials: γ increases by one unit the "pure ghost number" denoted *puregh*, while δ decreases the "antighost number" *antigh* by one unit. The antighost number is also called "antifield number". The ghost number *gh* is defined by

$$\mathrm{gh} = \mathrm{puregh} - \mathrm{antigh}\,. \tag{3.7}$$

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Z	$puregh(Z)$	$antigh(Z)$	$gh(Z)$	parity (mod 2)
$C_{\mu_1\ldots\mu_p\|\nu_1\ldots\nu_p}$	0	0	0	0
$A^{(i)}_{\mu_1\ldots\mu_p\|\nu_1\ldots\nu_{p-i}}$	i	0	i	i
$C^{*\mu_1\ldots\mu_p\|\nu_1\ldots\nu_p}$	0	1	-1	1
$A^{*(i)\mu_1\ldots\mu_p\|\nu_1\ldots\nu_{p-i}}$	0	$i+1$	$-(i+1)$	$i+1$

Table 1: Pureghost number, antighost number, ghost number and grassmannian parity of the fields.

Using the property $s^2 = 0$, one concludes that

$$\delta^2 = 0\,, \qquad \delta\gamma + \gamma\delta = 0\,, \qquad \gamma^2 = 0\,. \tag{3.8}$$

In the theories under consideration and according to the general rules of the BRST-antifield formalism, the spectrum of fields (including ghosts) and antifields is given by

- the field $C_{\mu_1\ldots\mu_p|\nu_1\ldots\nu_p}$, with ghost number zero and antifield number zero;
- the ghosts $A^{(i)}_{\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-i}}$ with ghost number i and antifield number zero, where $1 \leq i \leq p$;
- the antifield $C^{*\mu_1\ldots\mu_p|\nu_1\ldots\nu_p}$, with ghost number minus one and antifield number one;
- the antifields $A^{*(i)\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-i}}$ with ghost number minus $(i+1)$ and antifield number $(i+1)$, where $1 \leq i \leq p$.

The pureghost number, antighost number, ghost number and grassmannian parity of the various fields are displayed in table 1.

The action of the differentials δ and γ is zero on all the fields of the formalism except in the following cases:

$$\begin{aligned}
\delta C^{*\mu_1\ldots\mu_p|\nu_1\ldots\nu_p} &= G^{\mu_1\ldots\mu_p|\nu_1\ldots\nu_p} \\
\delta A^{*(1)\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-1}} &= -2\partial_\sigma C^{*\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-1}\sigma} \\
\delta A^{*(i)\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-i}} &= (-)^i \partial_\sigma A^{*(i-1)\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-i}\sigma}\,, \qquad 2 \leq i \leq p
\end{aligned}$$

and

$$\begin{aligned}
\gamma C_{\mu_1\ldots\mu_p|\nu_1\ldots\nu_p} &= A^{(1)}_{\mu_1\ldots\mu_p|[\nu_1\ldots\nu_{p-1},\nu_p]} + A^{(1)}_{\nu_1\ldots\nu_p|[\mu_1\ldots\mu_{p-1},\mu_p]} \\
\gamma A^{(i)}_{\mu_1\ldots\mu_p|\nu_1\ldots\nu_{p-i}} &= A^{(i+1)}_{\mu_1\ldots\mu_p|[\nu_1\ldots\nu_{p-i-1},\nu_{p-i}]} \\
&+ \ \epsilon(p,i) A^{(i+1)}_{\nu_1\ldots\nu_{p-i}[\mu_{p-i+1}\ldots\mu_p|\mu_1\ldots\mu_{p-i-1},\mu_{p-i}]}\,, \qquad 1 \leq i \leq p-1\,,
\end{aligned}$$

where $\epsilon(p,i) \equiv \frac{p!}{(i+1)!(p-i)!}$.

3.2 Standard cohomological results

The knowledge of $H^{D,0}(s|d)$ requires the computation of the following cohomological groups: $H(\gamma)$, $H(\gamma|d)$ in strictly positive antighost number, $H(\delta|d)$ and $H^{inv}(\delta|d)$. Some of these are already known [18, 19] and will be given in lemmas, whereas some have to be computed, which we do in the following theorems.

3.2.1 Cohomology of γ

The cohomology of γ (space of solutions of $\gamma a = 0$ modulo trivial coboundaries of the form γb) must be explicitly worked out and turns out to be isomorphic to the space of functions of the following variables:

- the antifields and all their derivatives, denoted by $[\Phi^*]$,
- the undifferentiated ghost $A^{(p)}_{[\mu_1\dots\mu_p]}$,
- the following undifferentiated "field strength" of the ghost $A^{(p)}$:$H^A_{[\mu_0\dots\mu_p]}\equiv\partial_{[\mu_0}A^{(p)}_{\mu_1\dots\mu_p]}$,
- the curvature K defined in (2.5) and all its derivatives, denoted by $[K]$.

The ghost-independent polynomials $\alpha([K],[\Phi^*])$ are called "invariant polynomials".

Comments. Let $\{\omega^I(A^{(p)}_{[\mu_1\dots\mu_p]}, H^A_{[\mu_0\dots\mu_p]})\}$ be a basis of the algebra of polynomials in the variables $A^{(p)}_{[\mu_1\dots\mu_p]}$ and $H^A_{[\mu_0\dots\mu_p]}$. Any element of $H(\gamma)$ can be decomposed in this basis, hence for any γ-cocycle α

$$\gamma\alpha = 0 \quad\Leftrightarrow\quad \alpha = \alpha_I([K_{\mu_1\dots\mu_{p+1}|\nu_1\dots\nu_{p+1}}],[\Phi^*])\,\omega^I\left(A^{(p)}_{[\mu_1\dots\mu_p]}, H^A_{[\mu_0\dots\mu_p]}\right) + \gamma\beta \qquad (3.9)$$

where the α_I are invariant polynomials. Furthermore, $\alpha_I\omega^I$ is γ-exact if and only if all the coefficients α_I are zero

$$\alpha_I\omega^I = \gamma\beta\,, \quad\Leftrightarrow\quad \alpha_I = 0\,, \qquad \text{for all } I\,. \qquad (3.10)$$

It should also be noted that, as $p>1$, there is no ghost in $H(\gamma)$ with pureghost number $1\leq k<p$, nor is there a ghost in $H(\gamma)$ with pureghost number $p+1$.

3.2.2 General properties of $H(\gamma|d)$

The cohomological space $H(\gamma|d)$ is the space of equivalence classes of forms a such that $\gamma a + db = 0$, identified by the relation $a\sim a' \Leftrightarrow a' = a+\gamma c + df$. We shall need properties of $H(\gamma|d)$ in strictly positive antighost (= antifield) number. To that end, we first recall the following result on invariant polynomials (pure ghost number = 0):

Lemma 1. *In form degree less than D and in antifield number strictly greater than 0, the cohomology of d is trivial in the space of invariant polynomials.*

See [17].

Lemma 1, which deals with d-closed invariant polynomials that involve no ghosts (one considers only invariant polynomials), has the following useful consequence on general γ-mod-d-cocycles with *antigh* >0, but possibly *puregh* $\neq 0$.

Consequence of lemma 1. If a has strictly positive antifield number (and involves possibly the ghosts), the equation $\gamma a + db = 0$ is equivalent, up to trivial redefinitions, to $\gamma a = 0$. That is,

$$\left.\begin{array}{r}\gamma a + db = 0\,,\\ antigh(a) > 0\end{array}\right\} \quad\Leftrightarrow\quad \left\{\begin{array}{l}\gamma a' = 0\,,\\ \;a' = a + dc\end{array}\right. . \qquad (3.11)$$

Thus, in antighost number >0, one can always choose representatives of $H(\gamma|d)$ that are strictly annihilated by γ. See [18, 19].

3.2.3 Characteristic cohomology $H(\delta|d)$

We now turn to the groups $H(\delta|d)$, i.e., to the solutions of the condition $\delta a + db = 0$ modulo trivial solutions of the form $a = \delta m + dn$.

Lemma 2. *The cohomology groups $H^D_q(\delta|d)$ vanish in antifield number q strictly greater than $p+1$,*

$$H^D_q(\delta|d) = 0 \qquad \text{for } q > p+1\,.$$

See [18, 19].

Theorem 1. *A complete set of representatives of $H^D_{p+1}(\delta|d)$ is given by the antifields $A^{*(p)[\mu_1\dots\mu_p]}$, i.e.,*

$$\begin{aligned}\delta a^D_{p+1} + da^{D-1}_p &= 0\\ \Rightarrow a^D_{p+1} &= \lambda_{[\mu_1\dots\mu_p]}A^{*(p)[\mu_1\dots\mu_p]}dx^0\wedge dx^1\wedge\dots\wedge dx^{D-1} + \delta b^D_{p+2} + db^{D-1}_{p+1}\end{aligned}$$

where the $\lambda_{[\mu_1\dots\mu_p]}$ are constants.

The proof runs as in the particular case $p=1$ treated in ref. [15] and will not be repeated here. Representatives with an explicit x-dependence were not considered, since they would not lead to Poincaré-invariant deformations.

Theorem 2. *The cohomology group $H^D_p(\delta|d)$ vanishes:*

$$\delta a^D_p + da^{D-1}_{p-1} = 0 \Rightarrow a^D_p = \delta b^D_{p+1} + db^{D-1}_p\,.$$

As far as theorem 2 is concerned, statements have been made recently in the particular case $p=2$ [11], the proof of which is absent and which are actually inaccurate. We therefore assume it is not irrelevant to give the complete proof of this theorem.

Proof of theorem 2: $H^D_p(\delta|d)$ is defined by

$$H^D_p(\delta|d) \cong \{a^D_p \quad | \quad \delta a^D_p + db^{D-1}_{p-1} = 0\,, \quad a^D_p \sim a^D_p + \delta\mu^D_{p+1} + d\mu^{D-1}_p\}\,.$$

The most general representative is

$$a^D_p = A^{*(p-1)}_{\nu_1\dots\nu_p|\nu_{p+1}}\Sigma^{\nu_1\dots\nu_p|\nu_{p+1}}d^Dx + \mu^D_p + \delta\mu^D_{p+1} + d\mu^{D-1}_p\,, \tag{3.12}$$

where μ^D_p is quadratic or more in the antifields. Acting on a^D_p with the Koszul-Tate differential yields

$$\delta a^D_p = (-)^{p-1}\partial_\sigma A^{*(p-2)\mu_1\dots\mu_p|\mu_{p+1}\sigma}\Sigma_{\mu_1\dots\mu_p|\mu_{p+1}}d^Dx + \delta\mu^D_p - d\delta\mu^{D-1}_p\,. \tag{3.13}$$

Taking the Euler-Lagrange derivative with respect to $A^{*(p-2)\mu_1\dots\mu_p|\mu_{p+1}\sigma}$ and demanding that a^D_p should belong to $H^D_p(\delta|d)$ gives the weak equation[2]

$$\mathbf{Y}^{[p,2]}\Sigma_{\mu_1\dots\mu_p|\nu_1,\nu_2} \approx 0\,, \tag{3.14}$$

[2] A weak equality denotes an equality up to terms that vanish on the surface of the solutions of the equations of motion.

where $\mathbf{Y}^{[\mathbf{p},\mathbf{2}]}$ projects on the Young-tableau symmetry $[p,2]$. Symbolically, the equation (3.14) reads

$$d^{\{2\}}\Sigma \approx 0\,, \tag{3.15}$$

where $d^{\{2\}}$ acts on the complex $\Omega_{(2)}(\mathbb{R}^D)$ [4]. Diagrammatically, the action of $d^{\{2\}}$ on the $[p,1]$-type tensor Σ looks like

$$d^{\{2\}}\ \begin{array}{|c|c|}\hline 1 & p{+}1 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} \vdots & \multicolumn{1}{c}{} \\ \cline{1-1} p & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \sim \begin{array}{|c|c|}\hline 1 & p{+}1 \\ \hline 2 & \partial \\ \hline \vdots & \multicolumn{1}{c}{} \\ \cline{1-1} p & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array}\ . \tag{3.16}$$

One defines the action of $d^{\{1\}}$ similarly [4]:

$$d^{\{1\}}\ \begin{array}{|c|c|}\hline 1 & \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} \vdots & \multicolumn{1}{c}{} \\ \cline{1-1} p & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \sim \begin{array}{|c|c|}\hline 1 & \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} \vdots & \multicolumn{1}{c}{} \\ \cline{1-1} p & \multicolumn{1}{c}{} \\ \cline{1-1} \partial & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array}\ . \tag{3.17}$$

On the other hand, the differentials d_1 and d_2 on a $[p,q]$-type tensor T are defined as follows:

$$\begin{aligned}(d_1T)_{\alpha\mu_1\ldots\mu_p|\nu_1\ldots\nu_q} &\equiv \partial_{[\alpha}T_{\mu_1\ldots\mu_p]|\nu_1\ldots\nu_q}\,,\\ (d_2T)_{\mu_1\ldots\mu_p|\nu_1\ldots\nu_q\alpha} &\equiv T_{\mu_1\ldots\mu_p|[\nu_1\ldots\nu_q,\alpha]}\,.\end{aligned} \tag{3.18}$$

Operating with $d^{\{1\}}$ on equation (3.14) we have $d^{\{1\}}d^{\{2\}}\Sigma \approx 0$ or $d_1d_2\Sigma \approx 0$, since $d^{\{1\}}d^{\{2\}} \equiv d_1d_2$ in $\Omega_{(2)}(\mathbb{R}^D)$ [4]. Considering the second column of the latter equation and using the isomorphism [18, 19] $H_0^1(d|\delta) \cong H_{D-1}^D(\delta|d) \cong 0$ where the last isomorphism holds because $D > p+2$, we have, in component: $\partial_{[\rho}\Sigma_{\mu_1\ldots\mu_p]|\nu_1} \approx \partial_{\nu_1}B_{\mu_1\ldots\mu_p\rho}$, where $B_{\mu_1\ldots\mu_p\rho}$ is a tensor completely antisymmetric in its $p+1$ indices. Antisymmetrizing the above equation in all its indices gives $0 \approx \partial_{[\nu}B_{\mu_1\ldots\mu_p\rho]}$ which defines a representative of the group $H_0^{p+1}(d|\delta)$, itself isomorphic to $H_{D-p-1}^D(\delta|d)$. The latter group vanishes for $D > 2p+2$ and is given by constants when $D = 2p+2$. Consequently, one has $B_{\mu_1\ldots\mu_p\rho} \approx D_{\mu_1\ldots\mu_p\rho}\delta_{2p+2}^D + \partial_{[\rho}C_{\mu_1\ldots\mu_p]}$ and $\partial_{[\rho}\Sigma_{\mu_1\ldots\mu_p]|\nu} \approx \partial_\nu\partial_{[\rho}C_{\mu_1\ldots\mu_p]}$. Solving this equation using $H_0^p(d|\delta) \cong H_{D-p}^D(\delta|d) \cong 0$ for $D > 2p+1$ gives $\Sigma_{\mu_1\ldots\mu_p|\nu} - C_{\mu_1\ldots\mu_p,\nu} \approx \partial_{[\mu_1}M_{\mu_2\ldots\mu_p]|\nu}$ for a given $M_{\mu_2\ldots\mu_p|\nu}$ completely antisymmetric in its first p indices. Its irreducible decomposition under $GL(D)$ consists in a sum of two types of fields: $M_{\mu_2\ldots\mu_p|\nu} \equiv M^A_{\mu_2\ldots\mu_p\nu} + M^B_{\mu_2\ldots\mu_p|\nu}$ where M^A is completely antisymmetric and M^B is of symmetry type $[p,1]$. Antisymmetrizing the above equation relating Σ, C and M yields (Σ and M^B both vanish due to their symmetry properties) $-C_{[\mu_1\ldots\mu_p,\nu]} \approx \partial_{[\mu_1}M^A_{\mu_2\ldots\mu_p\nu]}$. The latter equation, once solved, gives $M^A_{\mu_2\ldots\mu_p\mu_1} \approx C_{\mu_1\ldots\mu_p} + \partial_{[\mu_1}S_{\mu_2\ldots\mu_p]}$. The tensor Σ then decomposes as

$$\Sigma_{\mu_1\ldots\mu_p|\nu} \approx C_{\mu_1\ldots\mu_p,\nu} + \partial_{[\mu_1}M^B_{\mu_2\ldots\mu_p]|\nu} + C_{\nu[\mu_2\ldots\mu_p,\mu_1]}\,, \tag{3.19}$$

where S has been eliminated by redefining M^B. Substituting this decomposition back into (3.14) brings constraints on C and M^B. After some algebra, one finds

$\partial_{[\nu_1} M^B_{\nu_2\ldots\nu_p]|[\nu_{p+1},\nu_{p+2}]} \approx 0$. Again, considering the second column and using the isomorphism $H_0^1(d|\delta) \cong H^D_{D-1}(\delta|d) \cong 0$ (remember that $D > 2p+2$), we write $\partial_{[\nu_1} M^B_{\nu_2\ldots\nu_p]|\nu_{p+1}} \approx \partial_{\nu_{p+1}} B_{[\nu_1\ldots\nu_p]}$. Antisymmetrizing the latter equation and using $H_0^p(d|\delta) \cong H^D_{D-p}(\delta|d) \cong 0$ $(D > 2p+1)$, we obtain $\partial_{[\nu_1} M^B_{\nu_2\ldots\nu_p]|\nu_{p+1}} \approx \partial_{\nu_{p+1}} \partial_{[\nu_1} N_{\nu_2\ldots\nu_p]}$. Defining $\tilde{C}_{\nu_1\ldots\nu_p} = C_{\nu_1\ldots\nu_p} + \partial_{[\nu_1} N_{\nu_2\ldots\nu_p]}$ we finally get $\Sigma_{\mu_1\ldots\mu_p} \approx \tilde{C}_{\mu_1\ldots\mu_p,\nu} + \tilde{C}_{\nu[\mu_2\ldots\mu_p,\mu_1]}$.

As a result, a_p^D in eq. (3.12) reads

$$\begin{aligned} a_p^D &= (\tilde{C}_{\mu_1\ldots\mu_p,\nu} + \tilde{C}_{\nu\mu_2\ldots\mu_p,\mu_1}) A^{*(p-1)\mu_1\ldots\mu_p|\nu} d^D x + \mu_p^D + \delta\mu_{p+1}^D + d\mu_p^{D-1} \\ &= \frac{(p+1)}{p} \tilde{C}_{\mu_1\ldots\mu_p,\nu} A^{*(p-1)\mu_1\ldots\mu_p|\nu} d^D x + \mu_p^D + \delta\mu_{p+1}^D + d\mu_p^{D-1} \\ &= -\frac{(p+1)}{p} \tilde{C}_{\mu_1\ldots\mu_p} \partial_\nu A^{*(p-1)\mu_1\ldots\mu_p|\nu} d^D x + \mu_p^D + \delta\mu_{p+1}^D + d\mu'^{D-1}_p \,. \end{aligned} \tag{3.20}$$

Since $\delta A^{*(p)\mu_1\ldots\mu_p} = (-)^p \partial_\nu A^{*(p-1)\mu_1\ldots\mu_p|\nu}$, we see that only non-trivial part of a_p^D must be in the term μ_p^D which is quadratic or more in the antifields. However, such representatives of $H_p^D(\delta|d)$ are trivial [18], which implies $H_p^D(\delta|d) \cong 0$ in the theories under consideration. This proves theorem 2.

3.2.4 Invariant characteristic cohomology: $H^{inv}(\delta|d)$

The crucial result that underlies all consistent interactions does not deal with the general cohomology of δ modulo d but rather with the *invariant* cohomology of δ modulo d. The group $H^{inv}(\delta|d)$ is important because it controls the obstructions to removing the antifields from a s-cocycle modulo d [18, 19].

The central theorem that gives $H^{inv}(\delta|d)$ in antighost number $\geq p$ is

Theorem 3. *Assume that the invariant polynomial a_k^D ($k =$ antifield number) is δ-trivial modulo d,*

$$a_k^D = \delta\mu_{k+1}^D + d\mu_k^{D-1} \qquad (k \geq p)\,. \tag{3.21}$$

Then, one can always choose μ_{k+1}^D and μ_k^{D-1} to be invariant.

The proof of this theorem proceeds exactly as the proofs of similar theorems established for e.g. vector fields [19] or gravity [15]. Notice however that there is a subtlety one should not overlook in the case $k = p$; the appendix is devoted to clarify this specific issue in the particular case $p = 2$ (to fix the ideas). For $p > 2$, the proof follows exactly the same lines.

As a consequence of theorem 3, we have $H_k^{D,inv}(\delta|d) = 0$ for $k > p+1$, while $H_{p+1}^{D,inv}(\delta|d) = H_{p+1}^D(\delta|d)$ is given by theorem 1 and $H_p^{D,inv}(\delta|d)$ vanishes, by theorem 2.

3.3 First order consistent interactions

We can now proceed with the derivation of the cohomology of s modulo d in form degree D and in ghost number zero. A cocycle a of $H^{0,D}(s|d)$ must obey

$$sa + db = 0\,. \tag{3.22}$$

To analyze (3.22), we expand a and b according to the antifield number, $a = a_0 + a_1 + \cdots + a_k$, $b = b_0 + b_1 + \cdots + b_k$, where locality implies that the expansion stops at some finite antifield number [18, 19]. We recall [16] (i) that the antifield-independent piece a_0 is the deformation of the lagrangian; (ii) that a_1 contains the informations about the deformation of the gauge transformations; (iii) that a_2 contains the informations about the deformation of the gauge algebra and of the first-stage reducibility conditions; and (iv) that the a_k ($k \geq 3$) give the informations about the higher-stage reducibility conditions and the deformation of the higher order structure functions, which appear only when the algebra does not close off-shell. Thus, if one can show that the most general solution a of (3.22) stops at a_0, one can conclude that the gauge algebra is rigid and that the gauge transformations are not deformed at first order.

Writing s as the sum of γ and δ, the equation $sa + db = 0$ is equivalent to the system of equations $\delta a_i + \gamma a_{i-1} + db_{i-1} = 0$ for $i = 1, \dots, k$, and $\gamma a_k + db_k = 0$.

3.3.1 Computation of a_k, $k > p+1$

Following a general procedure used e.g. in [15], one can show that the terms a_k ($k > p+1$) may be discarded one after another from the aforementioned descent. The latter terminates thus with $\gamma a_{p+1} + db_{p+1} = 0$. Using the consequence of lemma 1, this equation is equivalent to $\gamma a_{p+1} = 0$.

Note that this result is independent of any condition on the number of derivatives or of Poincaré invariance. These requirements have not been used so far. The crucial ingredient of the proof is that the cohomological groups $H_k^{inv}(\delta|d)$, which control the obstructions to remove a_k from a, vanishes for $k > p+1$.

3.3.2 Computation of a_{p+1}

We thus have the following descent:

$$\delta a_1 + \gamma a_0 + db_0 = 0\,, \tag{3.23}$$

$$\vdots$$

$$\delta a_{p+1} + \gamma a_p + db_p = 0\,, \tag{3.24}$$

$$\gamma a_{p+1} = 0\,. \tag{3.25}$$

The last equation implies $a_{p+1} = \alpha_I \omega^I$. Acting with γ on the second to last equation and using lemma 1 leads to $b_p = \beta_I \omega^I$. Substituting those expressions in eq. (3.24), we find that a necessary (but not sufficient) condition for a_{p+1} to be a non-trivial solution of (3.24), such that a_p exists, is that α_I be a non-trivial element of $H^D_{p+1}(\delta|d)$. Asking for Poincaré invariance, theorem 1 then imposes $\alpha_I \sim A^{*(p)[\mu_1 \cdots \mu_p]}$. We next have to complete this α_I with an ω^I of ghost number $p+1$ in order to build a ghost zero candidate $\alpha_I \omega^I$ for a_{p+1}. However, there is no ω^I with pureghost number $p+1$, thus a_{p+1} must be zero.

3.3.3 Computation of a_k, $0 < k < p+1$

Following the same steps as for a_{p+1}, a_p writes $a_p = \alpha_I \omega^I$, where α_I belongs to $H^D_p(\delta|d)$. As shown in theorem 2, this group vanishes. Using theorem 3, it can be shown that one is allowed to set $a_p = 0$ (see [16]).

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Continuing the analysis, we have to find a_{p-1} of the form $a_{p-1} = \alpha_I \omega^I$, where α_I is a non-trivial element of $H^D_{p-1}(\delta|d)$. The ghost number of ω^I must be $p-1$, but there is no such ω^I, so a_{p-1} vanishes. One can repeat this argument for antifield number $k-2$, etc. until one reaches antifield number 0, where the argument does not work anymore.

At this point, we have proved the rigidity of the gauge algebra and gauge transformations. This has been done under the sole assumptions of locality and Poincaré invariance (besides the smoothness in the deformation parameter).

3.3.4 Computation of a_0

We are now reduced to solve the equation $\gamma a_0 + db_0 = 0$ for a_0. Such an a_0 corresponds to deformations of the lagrangian which are gauge invariant up to a total derivative. The Euler-Lagrange derivatives $\frac{\delta a_0}{\delta C}$ must be gauge invariant and must satisfy Bianchi identities of the type (2.8) (because of the gauge invariance of $\int a_0$). Asking that a_0 should not contain more than two derivatives, we obtain that $\frac{\delta a_0}{\delta C}$ must be at most linear in the curvature K. These three conditions together completely constrain a_0 and have only two solutions. The first one is a cosmological-constant-like term

$$a_0 = \Lambda \eta_{\mu_1\nu_1} \dots \eta_{\mu_p\nu_p} C^{\mu_1\dots\mu_p|\nu_1\dots\nu_p} \,. \tag{3.26}$$

The second one, where $\delta a_0/\delta C$ are linear in the curvature K, is the free lagrangian itself [8].

So we conclude that, apart from a cosmological-constant-like term, the deformation only changes the coefficient of the free lagrangian and is not essential.

4. Comments and conclusions

We can summarize our results as follows: under the hypothesis of locality and Poincaré invariance, there is no smooth deformation of the free theory which modifies the gauge algebra or the gauge transformations. If one further excludes deformations involving more than two derivatives in the lagrangian, then the only smooth deformation of the free theory is a cosmological-constant-like term.

Without this extra condition on the derivative order, one can introduce Born-Infeld-like interactions that involve powers of the gauge-invariant curvatures K. Such deformations modify neither the gauge algebra nor the gauge transformations.

We believe to have reached a fairly high degree of generality, these results being obtained under very few assumptions and constraining a whole class of theories for the gauge fields $C_{\mu_1\dots\mu_p|\nu_1\dots\nu_p}$, with p greater than one but otherwise arbitrary. The next step would be to consider arbitrary exotic "spin-2" fields represented by Young tableaux having two columns of different arbitrary lengths. It is planned to return to this question in the future.

Acknowledgments

We thank X. Bekaert for his comments. S.C. is grateful to M. Henneaux for enlightening discussions. The work of N.B. is supported by a Wiener-Anspach fellowship (Belgium), while the work of S.C. is supported in part by the "Actions de Recherche Concertées" of the

"Direction de la Recherche Scientifique - Communauté Française de Belgique", by a "Pôle d'Attraction Interuniversitaire" (Belgium), by IISN-Belgium (convention 4.4505.86) and by the European Commission RTN program HPRN-CT-00131, in which she is associated to K. U. Leuven.

A. Proof of theorem 3 for $k = p$

As we said, the proof of theorem 3 proceeds as the proof of similar theorems established for e.g. vector field [19] or gravity [15].

At some stage one must replace the fields appearing on the right-hand-side of the expression for the Euler-Lagrange derivatives of an invariant a_k (see eqs. (6.6)–(6.10) of ref. [19]) by invariant quantities. Usually, one first shows that this can be done for the first equation (the analogue of eq. (6.6) of ref. [19]), then the property propagates to all the other equations. However, for $k = p$, the first equation is trivially satisfied and one must therefore start with the second one, which in our case possesses the following structure:

$$\frac{\delta a_p}{\delta A^{*(p-1)}} = \delta Z_1 + d^{\{2\}} W_0 \,. \tag{A.1}$$

To summarize, what one has to do in the case $k = p$ is the following: using only the above equation, to show that both Z and W can be taken to be invariant quantities. In the sequel we consider $p = 2$ to fix the ideas, but the general p-arbitrary case proceeds exactly in the same way.

Theorem 4. *Let the zero-antighost-number invariant polynomial α_0 satisfy*

$$\alpha_{0\,\mu\nu|\rho} = \delta Z_{1\,\mu\nu|\rho} + (\partial_\rho W_{0\,\mu\nu} - \partial_{[\rho} W_{0\,\mu\nu]}) \,, \tag{A.2}$$

where α_0 and Z_1 are [2,1]-tensors and W_0 is a [2,0]-tensor, then

$$\begin{aligned} Z_{1\,\mu\nu|\rho} &= Z'_{1\,\mu\nu|\rho} + \delta\phi_{2\,\mu\nu|\rho} + (\partial_\rho \chi_{1\,[\mu\nu]} - \partial_{[\rho}\chi_{1\,\mu\nu]}) \,, && \text{(A.3)} \\ W_{0\,\mu\nu} &= W'_{0\,[\mu\nu]} + \delta\chi_{1\,[\mu\nu]} \,, && \text{(A.4)} \end{aligned}$$

for some invariants Z'_1 and W'_0.

To prove this theorem, we need the following lemma:

Lemma 3. *Suppose that theorem 3 is proved for $k > p$. If α_0^1 is an invariant polynomial and satisfies*

$$\alpha_0^1 = \delta Z_1^1 + dW_0^0 \,, \tag{A.5}$$

then, for some invariant polynomials ${Z'}_1^1$ and ${W'}_0^0$,

$$\begin{aligned} Z_1^1 &= {Z'}_1^1 + \delta\phi_2^1 + d\chi_1^0 \,, && \text{(A.6)} \\ W_0^0 &= {W'}_0^0 + \delta\chi_1^0 \,. && \text{(A.7)} \end{aligned}$$

Though lemma 3 is already known [20, 6], the proof does never appear explicitely. We shall therefore give it here.

Using standard techniques, one gets the following descent

$$\begin{aligned} \alpha_1^2 &= \delta Z_2^2 + dZ_1^1 \\ &\vdots \\ \alpha_{D-1}^D &= \delta Z_D^D + dZ_{D-1}^{D-1}, \end{aligned} \tag{A.8}$$

where all the α_{i-1}^i are invariant. As $D-1 \geq p+1$, by theorem 3 we can choose Z_D^D and Z_{D-1}^{D-1} invariant. The invariance property propagates up until $\alpha_1^2 = \delta Z'^2_2 + dZ'^1_1$, where Z'^2_2 and Z'^1_1 have been chosen invariant. Substracting the latter equation from (A.8) and knowing that $H_1^1(\delta|d) \cong H_D^D(\delta|d)$ vanishes, we get (A.6). Substituting (A.6) in (A.5) and acting with γ, we find $d(\gamma(W_0^0 - \delta\chi_1^0)) = 0$. Using the algebraic Poincaré lemma and the fact that there is no constant with positive pureghost number, this implies $\gamma(W_0^0 - \delta\chi_1^0) = 0$, which in turn gives (A.7), as there exists no γ-exact term of pureghost number 0.

Proof of theorem 4: the first step is to constrain the last term of (A.2). In (hyper)form notation, eq. (A.2) reads:

$$\alpha_0^{[2,1]} = \delta Z_1^{[2,1]} + d^{\{2\}} W_0^{[2,0]}. \tag{A.9}$$

Acting with $d^{\{1\}}$ on eq. (A.9), we get

$$d^{\{1\}}\alpha_0^{[2,1]} = \delta(d^{\{1\}} Z_1^{[2,1]}) + d^{\{2\}}(d^{\{1\}} W_0^{[2,0]}). \tag{A.10}$$

We now consider only the second column. Since $d^{\{2\}}d^{\{1\}}W_0^{[2,0]} \equiv d_2 d_1 W_0^{[2,0]}$, Equation (A.10) reads

$$\tilde{\alpha}_0^1 = \delta\tilde{Z}_1^1 + d\tilde{W}_0^0. \tag{A.11}$$

As $\tilde{\alpha}_0^1$ is invariant, we know by lemma 3 that $\tilde{W}_0^0 = \tilde{W}'^0_0 + \delta\beta_1^0$, where $\tilde{W}'^0_0$ is invariant. Remembering that $\tilde{W}_0^0 = d_1 W_0^{[2,0]}$, we have in components: $\partial_{[\sigma} W_{0\,\mu\nu]} = \tilde{W}'_{0\,[\mu\nu\sigma]} + \delta\beta_{1\,[\mu\nu\sigma]}$ for some invariant $\tilde{W}'_{0\,[\mu\nu\rho]}$. Plugging this in (A.2), we get

$$\alpha_{0\,\mu\nu|\rho} + \tilde{W}'_{0\,[\mu\nu\rho]} = \delta(Z_{1\,\mu\nu|\rho} - \beta_{1\,[\mu\nu\rho]}) + \partial_\rho W_{0\,\mu\nu}. \tag{A.12}$$

Considering only ρ as a form index, we have

$$\bar{\alpha}^1_{0|[\mu\nu]} = \delta\bar{Z}^1_{1|[\mu\nu]} + dW^0_{0|[\mu\nu]}, \tag{A.13}$$

which, as $\bar{\alpha}^1_{0|\mu\nu}$ is invariant, implies that $\bar{Z}^1_{1|\mu\nu} = \bar{Z}'^1_{1|\mu\nu} + \delta\chi^1_{2|\mu\nu} + d\chi^0_{1|\mu\nu}$ and $W^0_{0|\mu\nu} = W'^0_{0|\mu\nu} + \delta\chi^0_{1|\mu\nu}$, for some invariants $\bar{Z}'^1_{1|\mu\nu}$ and $W'^0_{0|\mu\nu}$. Equivalently, we have

$$Z_{1\,\mu\nu|\rho} = \beta_{1\,[\mu\nu\rho]} + \bar{Z}'_{1\rho||[\mu\nu]} + \delta\chi_{2\,\rho||[\mu\nu]} + \partial_\rho\chi_{1\,[\mu\nu]}, \tag{A.14}$$

$$W_{0\,\mu\nu} = W'_{0\,[\mu\nu]} + \delta\chi_{1\,[\mu\nu]}. \tag{A.15}$$

Removing the completely antisymmetric part from (A.14), we get the wanted result

$$Z_{1\,\mu\nu|\rho} = Z'_{1\,\mu\nu|\rho} + \delta\phi_{2\,\mu\nu|\rho} + (\partial_\rho\chi_{1\,[\mu\nu]} - \partial_{[\rho}\chi_{1\,\mu\nu]}), \tag{A.16}$$

where $Z'_{1\,\mu\nu|\rho} \equiv \bar{Z}'_{1\rho|[\mu\nu]} - \bar{Z}'_{1[\rho|\mu\nu]}$ and $\phi_{2\,\mu\nu|\rho} \equiv \chi_{2\,\rho|[\mu\nu]} - \chi_{2\,[\rho|\mu\nu]}$ are $[2,1]$-tensors, and Z'_1 is invariant.

We stress again that the proof in the case where p is arbitrary proceeds exactly in the same way.

The rest of the proof of theorem 3 follows exactly the lines explained in e.g. refs. [19] and [15].

References

[1] C. Burdik, A. Pashnev and M. Tsulaia, *On the mixed symmetry irreducible representations of the Poincaré group in the brst approach*, *Mod. Phys. Lett.* **A 16** (2001) 731 [hep-th/0101201]; *The lagrangian description of representations of the Poincaré group*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 285 [hep-th/0103143].

[2] H. Casini, R. Montemayor and L.F. Urrutia, *Dual theories for mixed symmetry fields. spin-two case: (1,1) versus (2,1) young symmetry type fields*, *Phys. Lett.* **B 507** (2001) 336 [hep-th/0102104].

[3] M. Dubois-Violette and M. Henneaux, *Tensor fields of mixed young symmetry type and N-complexes*, *Commun. Math. Phys.* **226** (2002) 393 [math.qa/0110088].

[4] X. Bekaert and N. Boulanger, *Tensor gauge fields in arbitrary representations of $GL(D, R)$: duality and Poincaré lemma*, *Commun. Math. Phys.* **245** (2004) 27 [hep-th/0208058].

[5] P. de Medeiros and C. Hull, *Exotic tensor gauge theory and duality*, *Commun. Math. Phys.* **235** (2003) 255 [hep-th/0208155].

[6] X. Bekaert, N. Boulanger and M. Henneaux, *Consistent deformations of dual formulations of linearized gravity: a no-go result*, *Phys. Rev.* **D 67** (2003) 044010 [hep-th/0210278].

[7] X. Bekaert and N. Boulanger, *On geometric equations and duality for free higher spins*, *Phys. Lett.* **B 561** (2003) 183 [hep-th/0301243]; *Mixed symmetry gauge fields in a flat background*, hep-th/0310209.

[8] P. de Medeiros and C. Hull, *Geometric second order field equations for general tensor gauge fields*, *J. High Energy Phys.* **05** (2003) 019 [hep-th/0303036].

[9] N. Boulanger, S. Cnockaert and M. Henneaux, *A note on spin-s duality*, *J. High Energy Phys.* **06** (2003) 060 [hep-th/0306023].

[10] Y.M. Zinoviev, *First order formalism for mixed symmetry tensor fields*, hep-th/0304067; *First order formalism for massive mixed symmetry tensor fields in minkowski and (A)ds spaces*, hep-th/0306292.

[11] C. Bizdadea, C.C. Ciobirca, E.M. Cioroianu, S.O. Saliu and S.C. Sararu, *Interactions of a massless tensor field with the mixed symmetry of the riemann tensor: no-go results*, hep-th/0306154.

[12] K.B. Alkalaev, O.V. Shaynkman and M.A. Vasiliev, *On the frame-like formulation of mixed-symmetry massless fields in (A)ds(d)*, hep-th/0311164;
M.A. Vasiliev, *Higher spin gauge theories in various dimensions*, hep-th/0401177;
K.B. Alkalaev, *Two-column higher spin massless fields in AdS(d)*, hep-th/0311212;
P. de Medeiros, *Massive gauge-invariant field theories on spaces of constant curvature*, hep-th/0311254.

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[13] G. Bonelli, *On the tensionless limit of bosonic strings, infinite symmetries and higher spins*, *Nucl. Phys.* **B 669** (2003) 159 [hep-th/0305155]; *On the covariant quantization of tensionless bosonic strings in AdS spacetime*, *J. High Energy Phys.* **11** (2003) 028 [hep-th/0309222].

[14] A. Sagnotti and M. Tsulaia, *On higher spins and the tensionless limit of string theory*, hep-th/0311257;
X. Bekaert, I.L. Buchbinder, A. Pashnev and M. Tsulaia, *On higher spin theory: strings, BRST, dimensional reductions*, hep-th/0312252.

[15] N. Boulanger, T. Damour, L. Gualtieri and M. Henneaux, *Inconsistency of interacting, multigraviton theories*, *Nucl. Phys.* **B 597** (2001) 127 [hep-th/0007220].

[16] G. Barnich and M. Henneaux, *Consistent couplings between fields with a gauge freedom and deformations of the master equation*, *Phys. Lett.* **B 311** (1993) 123 [hep-th/9304057];
M. Henneaux, *Consistent interactions between gauge fields: the cohomological approach*, *Contemp. Math.* **219** (1998) 93 [hep-th/9712226].

[17] M. Dubois-Violette, M. Henneaux, M. Talon and C.-M. Viallet, *Some results on local cohomologies in field theory*, *Phys. Lett.* **B 267** (1991) 81.

[18] G. Barnich, F. Brandt and M. Henneaux, *Local brst cohomology in the antifield formalism, 1. General theorems*, *Commun. Math. Phys.* **174** (1995) 57 [hep-th/9405109];

[19] G. Barnich, F. Brandt and M. Henneaux, *Local brst cohomology in the antifield formalism, ii. Application to Yang-Mills theory*, *Commun. Math. Phys.* **174** (1995) 93 [hep-th/9405194]; for a review, see *Local BRST cohomology in gauge theories*, *Phys. Rept.* **338** (2000) 439 [hep-th/0002245].

[20] M. Henneaux, B. Knaepen and C. Schomblond, *Characteristic cohomology of p-form gauge theories*, *Commun. Math. Phys.* **186** (1997) 137 [hep-th/9606181].